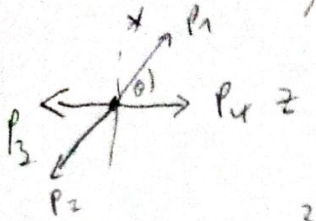
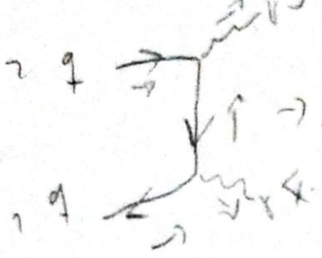
20 | 22.3 | $s = \sin\theta$ cos analog
$s' = \sin\frac{\theta}{2}$

$$M = \frac{(Qg)^2}{4p^2}\bar{V}(1)\Big[\Big(\frac{1}{s'^2}\big(\not p(4)\not\varepsilon^*(4)\not\varepsilon^*(3) + 2(p_1\,\varepsilon^*(4))\not\varepsilon(3)\big)\Big) + \Big(\frac{1}{c'^2}\big(\not p(3)\not\varepsilon^*(3)\not\varepsilon^*(4) + 2(p_1\varepsilon^*(3))\not\varepsilon(4)\big)\Big]$$

$-\not p(4)$



$\boxed{\varepsilon^*_\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\ i\\ 0\end{pmatrix} \quad \vec\varepsilon^*_\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix} \quad \vec\varepsilon^*_\uparrow\vec\varepsilon^*_\downarrow = -1 \quad \varepsilon^*_\uparrow \times \varepsilon^*_\downarrow = \begin{pmatrix}0\\0\\-i\end{pmatrix}}$



$$M_1 = \frac{(Qg)^2}{(p_1-p_4)^2}\bar V(1)\overbrace{\not\varepsilon(4)(\not p_1 - \not p_4)\not\varepsilon(3)}^{\Gamma_1}U(2)$$
$(p_1-p_4)^2 = 4p^2s'^2$

$$M_2 = \frac{(Qg)^2}{(p_1-p_3)^2}\bar V(1)\not\varepsilon(3)(\not p_1 - \underbrace{\not p_3}_{=+\not p_4})\not\varepsilon(4)U(2)$$
$(p_1-p_3)^2 = 4p^2 c'^2$

$\hookrightarrow$ ↯ Falsch !!

$U_\uparrow(2) = \sqrt{p}\begin{pmatrix}s'\\c'\\s'\\c'\end{pmatrix}$ $U_\downarrow(2) = \sqrt{p}\begin{pmatrix}-c'\\s'\\+c'\\-s'\end{pmatrix}$

$\not\ell_{180°\mathrm{rot}}$

$\bar V(1)_\uparrow = \sqrt{p}(s'\ -c'\ s'\ -c')$
$\bar V(1)_\downarrow = \sqrt{p}(c'\ s'\ -c'\ -s')$

$\bar V$	U	$\varepsilon(3)$	$\varepsilon(4)$
↑	↑	↑	↓
↑	↑	↓	↑
↓	↓	↑	↓
↓	↓	↓	↑

$$\Gamma_1 = \cancel{\not p_1}\,2\, p_1\varepsilon(4)\not\varepsilon(3) - \not p_1\not\varepsilon(4)\not\varepsilon(3) + \not p_4\not\varepsilon(4)\not\varepsilon(3)$$
$$\Gamma_2 = 2\, p_1\varepsilon(3)\not\varepsilon(4) - \not p_1\not\varepsilon(3)\not\varepsilon(4) + \not p_3\not\varepsilon(3)\not\varepsilon(4)$$
$= -\not p_4\not\varepsilon(3)\not\varepsilon(4)$

$\bar V(1)\not p_1 = 0$

$$\Rightarrow M_1 = \frac{(Qg)^2}{-4p^2s'^2}\bar V(1)\big(2(p_1\cdot\varepsilon(4))\not\varepsilon(3) + \not p(4)\not\varepsilon(4)\not\varepsilon(3)\big)U(2)$$

$$M_2 = \frac{(Qg)^2}{-4p^2c'^2}\bar V(1)\big(2(p_1\varepsilon(3))\not\varepsilon(4) - \cancel{\not p(4)}\not\varepsilon(3)\not\varepsilon(4)\big)U(2)$$
$+\not p(3)$! ⟵ F!

$\varepsilon^*_\uparrow\varepsilon^*_\uparrow = 0$

$\not\varepsilon^*_\uparrow = \sqrt{2}\begin{pmatrix} & & & 0\\ & & 1 & \\ & 0 & & \\ -1 & & & \end{pmatrix}$ $\not\varepsilon^*_\downarrow = \sqrt{2}\begin{pmatrix} & & 0 & -1\\ & & & \\ 0 & 1 & & \end{pmatrix}$

✓ $\boxed{p_1\varepsilon^*_\uparrow = \frac{ps}{\sqrt{2}} \quad p_1\varepsilon^*_\downarrow = \frac{-ps}{\sqrt{2}}}$

$\bar V\not\varepsilon^* U$: ↑↑↑ $= \sqrt{2}p\,(s'\ -c'\ s'\ -c')\begin{pmatrix}0\\s'\\0\\-s'\end{pmatrix}$
$= p\sqrt{2}(-c's' + c's') = 0$

↑↓↑ $= \sqrt{2}p\,(s'\ -c'\ s'\ -c')\begin{pmatrix}-c'\\0\\c'\\0\end{pmatrix} = 0$

↑↑↓ $= \sqrt{2}p\,(s'\ -c'\ s'\ -c')\begin{pmatrix}0\\c'\\0\\c'\end{pmatrix}$
$= \sqrt{2}p(-c'^2 - c'^2) = -2\sqrt{2}p\,c'^2$

~~$\frac{↑↓↓}{\sqrt{2}p} = (s'\ -c'\ s'\ -c')(\ldots) = 2sc'$~~

$\frac{↓↑↑}{\sqrt{2}p} = (c'\ s'\ -c'\ -s')\begin{pmatrix}0\\s'\\0\\-s'\end{pmatrix} = 2s'^2$

$\frac{↓↑↓}{\ldots} = (c'\ s'\ -c'\ -s')\begin{pmatrix}0\\c'\\0\\c'\end{pmatrix}$
$= 0$

$\downarrow\downarrow\uparrow = (c' s' -c' -s')\begin{pmatrix}-c\\0\\c\\0\end{pmatrix} = -2c'^2$ $\quad \underset{\sqrt{..}}{\uparrow\downarrow\downarrow} = (s' -c' s' -c')\begin{pmatrix}s'\\0\\s'\\0\end{pmatrix}$

$\downarrow\downarrow\downarrow = (c' s' -c' -s')\begin{pmatrix}s'\\0\\s'\\0\end{pmatrix} = 0$ $\quad = 2s'^2$

$\checkmark \neq 0 \to \underset{...}{\uparrow\uparrow\downarrow} = -2c'^2$ $\quad \underset{1\sqrt{2}P}{\uparrow\downarrow\downarrow} = \cancel{-2s'c' = -s} = 2s'^2$ $\quad \underset{..}{\downarrow\uparrow\uparrow} = 2s'^2$ $\quad \underset{..}{\downarrow\downarrow\uparrow} = -2c'^2$

↳ Trick: Symmetrie in Erg! + $\theta \to \theta + 180 \to$ umdrehen der Erg !! :)

$P_4 \psi^*(a)\psi^*(b)$: $\psi_\uparrow^*\psi_\uparrow^* = \psi_\downarrow^*\psi_\downarrow^* = 0!$ $\quad \underset{2}{\psi_\uparrow^*\psi_\downarrow^*} = \begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix}$

$\checkmark$ $\underset{2}{\varepsilon_\downarrow^*\varepsilon_\uparrow^*} = \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}$ $\quad \psi_\uparrow\psi_\downarrow^2 + \psi_\downarrow\psi_\uparrow^* = -2 \cdot -1 = 2\mathbb{1}$ ✓

$\checkmark$ $P_4 = P\begin{pmatrix}\mathbb{1} & \begin{matrix}-1&0\\0&1\end{matrix}\\ \begin{matrix}1&0\\0&-1\end{matrix} & -\mathbb{1}\end{pmatrix}$ $\quad P_4 \uparrow\downarrow = \begin{pmatrix}0&0&0&0\\0&1&0&1\\0&0&0&0\\0&-1&0&-1\end{pmatrix}$ $\quad P_4 \downarrow\uparrow = \begin{pmatrix}1&0&-1&0\\0&0&0&0\\1&0&-1&0\\0&0&0&0\end{pmatrix}$

$\bar{\psi} P_4 \psi^*\psi^* \psi$: $\cancel{\uparrow\downarrow\uparrow}$ $\quad \underset{2\sqrt{P}P}{\uparrow\downarrow\uparrow} = \begin{pmatrix}0\\2c'\\0\\-2c'\end{pmatrix}$ $\quad \uparrow\downarrow\downarrow = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ $\quad \downarrow\uparrow\uparrow = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ $\quad \downarrow\uparrow\downarrow = \begin{pmatrix}-2c'\\0\\-2c'\\0\end{pmatrix}$

$\underset{2P^2}{\uparrow\uparrow\downarrow\uparrow} = (s' -c' s' -c')\begin{pmatrix}0\\2c'\\0\\-2c'\end{pmatrix} = 0$ $\quad \underset{2P^2}{\uparrow\uparrow\downarrow\downarrow} = 0$ $\quad \underset{...}{\uparrow\downarrow\uparrow\uparrow} = 0$ $\quad \uparrow\downarrow\uparrow\downarrow = (s' -c' s' -c')(\ldots = -4c's' = -2s$

$\underset{...}{\downarrow\uparrow\downarrow\uparrow} = (c' s' -c' -s')\begin{pmatrix}0\\2c\\0\\-2c'\end{pmatrix} = 4s'c' = 2s$ $\quad \downarrow\uparrow\downarrow\downarrow = 0$ $\quad \downarrow\downarrow\uparrow\uparrow = 0$

$\checkmark$

$\underset{\text{sra}}{\downarrow\downarrow\uparrow\downarrow} = (c' s' -c' -s')\begin{pmatrix}-2c'\\0\\-2c'\\0\end{pmatrix} = 0$ $\quad \neq 0$: $\boxed{\uparrow\downarrow\uparrow\downarrow = -4P^2 s \quad \downarrow\uparrow\downarrow\uparrow = 4P^2 s}$

ok unter 180° drehung ✓

22.12.22.3

$4p^2 \frac{\mathcal{M}}{(e s)^2}$:

$$\frac{1}{s'^2}\left((p_1 \cdot \varepsilon)\, \bar{v}_4 \not{\varepsilon}_3 \not{\varepsilon}_2 u + \frac{1}{\bar v}\, p_{(1)} \not{\varepsilon} \not{\varepsilon} u \right) + \frac{1}{c'^2}\left(p_{(1)} \bar v_4 \not{\varepsilon}_3 \not{\varepsilon}_2 u - \bar v_4 \not{\varepsilon} \not{\varepsilon} u \right)$$

$$\underset{1342}{\uparrow\uparrow\uparrow\uparrow} = \frac{1}{s'^2}\left(\uparrow \cdot \uparrow\uparrow\uparrow + \uparrow\uparrow\uparrow\uparrow \right) + \frac{1}{c'^2}\left(\uparrow \cdot \uparrow\uparrow\uparrow - \uparrow\uparrow\uparrow \right)$$

$0 \qquad 0 \qquad 0$

$= \frac{1}{s^2}\left(0 \right)$

$= 0$

$$\underset{1234}{\uparrow\uparrow\uparrow\downarrow} = \frac{1}{s'^2}\left(\downarrow \cdot \uparrow\uparrow\uparrow + \uparrow\downarrow\uparrow\uparrow \right) + \frac{1}{c'^2}\left(\uparrow \cdot \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow\uparrow \right)$$

$= 0$

$$\underset{1234}{\uparrow\uparrow\downarrow\uparrow} = \frac{1}{s'^2}\left(\uparrow \cdot \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow\uparrow \right) + \frac{1}{c'^2}\left(\downarrow \cdot \uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow \right)$$

$0 \qquad 0 \qquad 0 \qquad 0$

$= 0$

$$\overset{1234}{\uparrow\uparrow\downarrow\downarrow} = \frac{1}{s'^2}\left(\downarrow \cdot \uparrow\downarrow\uparrow + \uparrow\downarrow\downarrow\uparrow \right) + \frac{1}{c'^2}\left(\downarrow \cdot \uparrow\downarrow\uparrow - \uparrow\downarrow\downarrow\uparrow \right)$$

$= 0$

$$\underset{1234}{\uparrow\downarrow\uparrow\uparrow} = \frac{1}{s'^2}\left(\uparrow \cdot \uparrow\uparrow\downarrow + \uparrow\uparrow\uparrow\downarrow \right) + \frac{1}{c'^2}\left(\uparrow \cdot \uparrow\uparrow\downarrow - \uparrow\uparrow\uparrow\downarrow \right)$$

$$= \frac{1}{s'^2} \cdot \left(-\frac{p^2 s}{\sqrt{2}} \sqrt{2}\, 2c'^2 \right) + \frac{1}{c'^2} \cdot \left(-p^2 s\, 2c'^2 \right) = -2 s c'^2 p^2 \left(\frac{1}{s'^2} + \frac{1}{c'^2} \right) = -\frac{c'^2}{s} p^2\, 8$$

$$= -2p^2 s \left(\frac{c'^2}{s'^2} + \frac{s'^2}{s'^2} \right) = -4p'^2 \left(\frac{c'}{s'} \right) = \underline{\underline{-4p^2 \cot'^2}}$$

$$\cancel{p^2 2s\left(\frac{c'^2}{s'^2}+1\right)} = -p^2 2s \frac{1}{s'^2 c'^2} = \frac{4}{s^2}$$

$$\underset{1234}{\uparrow\downarrow\uparrow\downarrow} = \frac{1}{s'^2}\left(\downarrow \cdot \uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow\downarrow \right) + \frac{1}{c'^2}\left(\uparrow \cdot \uparrow\downarrow\downarrow - \uparrow\uparrow\downarrow\downarrow \right)$$

$$= \frac{1}{s'^2}\left(+ \frac{p^2 s}{\sqrt{2}} \sqrt{2}\, 2c'^2 - 4p^2 s \right) + \frac{1}{c'^2}\left(\frac{p^2 s}{\sqrt{2}} \sqrt{2}\, 2s'^2 \right)$$ im Weg?

$$= -\frac{4p^2 s}{s'^2} + 2p^2 s \left(\frac{c'^2}{s'^2} + \frac{s'^2}{c'^2} \right) = \underline{\underline{2p^2 s \left((\cot'^2 + \tan'^2) - 2\frac{1}{s'^2} \right)}}$$

$= \tan'^2 - \cot'^2 - 2$

↳ ugly

$$\underset{1234}{\uparrow\downarrow\downarrow\uparrow} = \frac{1}{s'^2}\left(\uparrow \cdot \uparrow\downarrow\downarrow + \uparrow\uparrow\downarrow\downarrow \right) + \frac{1}{c'^2}\left(\downarrow \cdot \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow\downarrow \right)$$

$$= \frac{1}{s'^2}\left(\frac{p^2 s}{\sqrt{2}} \sqrt{2}\, 2s'^2 + 0 \right) + \frac{1}{c'^2}\left(+ p^2 s\, 2c'^2 + 4p^2 s \right)$$

$$= 4p^2 s \cancel{1} + \frac{4p^2 s}{c'^2} = \underline{\underline{4p^2 s \left(1 + \frac{1}{c'^2} \right)}} = 4p^2 s (2 + \tan'^2) = 4p^2 s + \frac{2 s'}{c'^2}$$

$$= \underline{\underline{4p^2 (s + 2\tan')}}$$

$1 + 1 + \tan'^2 = 2 + \tan'^2$

23 | 22.3

$\uparrow\downarrow\downarrow\downarrow$ (1 2 3 4) $= \frac{1}{s'^2}(\downarrow\uparrow\downarrow\downarrow + \uparrow\downarrow\downarrow\downarrow) + \frac{1}{c'^2}(\downarrow\uparrow\downarrow\downarrow - \uparrow\downarrow\downarrow\downarrow)$

$= \frac{1}{s'^2}\left(-\frac{p^2 s}{\sqrt{2}}\sqrt{2}\, 2s'^2 + 0\right) + \frac{1}{c'^2}\left(-p^2 s\, 2s'^2\right)$

$= -2p^2 s(1 + \tan'^2) = -2p^2\frac{s}{c'^2} = -4p^2\frac{s'c'}{c'^2} = \underline{\underline{-4p^2\tan'}}$

$\downarrow\uparrow\uparrow\uparrow$ (1 2 3 4) $= \frac{1}{s'^2}(\uparrow\downarrow\uparrow\uparrow + \downarrow\uparrow\uparrow\uparrow) + \frac{1}{c'^2}(\uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow)$

$= \frac{1}{s'^2}\left(\frac{p^2 s}{\sqrt{2}}\sqrt{2}\, 2s'^2\right) + \frac{1}{c'^2}p^2 s\, 2s'^2 = 2p^2 s(1 + \tan'^2)$

$\downarrow\uparrow\uparrow\downarrow$ (1 2 3 4) $= \frac{1}{s'^2}(\downarrow\downarrow\uparrow\uparrow + \downarrow\downarrow\uparrow\uparrow) + \frac{1}{c'^2}(\uparrow\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow\uparrow)$

$= \frac{1}{s'^2}(-p^2 s\, 2s'^2) + \frac{1}{c'^2}(-p^2 s\, 2c'^2 - 4p^2 s)$

$= \cancel{-2p^2 s + -4p^2 s = -}\ -4p^2 s\left(1 + \frac{1}{c'^2}\right)$

$\downarrow\uparrow\downarrow\uparrow$ (1 2 3 4) $= \frac{1}{s'^2}(\uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow) + \frac{1}{c'^2}(\downarrow\downarrow\uparrow\uparrow - \downarrow\downarrow\uparrow\uparrow)$

$= \frac{1}{s'^2}(-p^2 s\, 2c'^2 + 4p^2 s) + \frac{1}{c'^2}(-p^2 s\, 2s'^2)$

$= -2p^2 s\left(\frac{c'^2}{s'^2} + \frac{s'^2}{c'^2}\right) \ldots \frac{2}{s'^2})$

$\downarrow\downarrow\uparrow\uparrow = 0 \quad \downarrow\downarrow\uparrow\downarrow = 0 \quad \downarrow\downarrow\downarrow\uparrow = 0 \quad \downarrow\downarrow\downarrow\downarrow = 0$

$\downarrow\uparrow\downarrow\downarrow = +2p^2 s\left(\frac{c'^2}{s'^2} + 1\right)$

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$$s^{12}\bar{\Gamma}_1 = \gamma^0 (p_1 \mathcal{E}(4))^* \not{\varepsilon}(3)^\dagger + \cancel{\not{\varepsilon}(3)\not{\varepsilon}(4)} \quad \cancel{\not{p}(4)\not{\varepsilon}(4)\not{\varepsilon}(3))}$$
$$\not{\varepsilon}(3)^\dagger \not{\varepsilon}(4)^\dagger \not{p}(4)^\dagger)\, \gamma^0$$

$$= p_1 \mathcal{E}(\overset{a}{4})\; \not{\varepsilon}(\overset{b}{3}) + \not{\varepsilon}(3)\not{\varepsilon}(4)\not{p}(\overset{a}{4})$$

complex*

$a_1 = 4 \quad b_1 = 3$
$a_2 = 3 \quad b_2 = 4$

$$c^{12}\bar{\Gamma}_2 = p_1 \mathcal{E}(3)\, \not{\varepsilon}(4) \quad - \not{\varepsilon}(4)\not{\varepsilon}(3)\not{p}(4)$$

$$\not{p}(1) = -\not{p}(1)$$

$$|\mathcal{M}|^2 = -\left(\frac{e g)^2}{4p^2}\right)^2 \operatorname{tr}(\Gamma \not{p}(1)\, \bar{\Gamma}\, \not{p}(1))$$

$$(\Gamma_1 + \Gamma_2)\not{p}(\bar{\Gamma}_1 + \bar{\Gamma}_2)\not{p} = \Gamma_1 \not{p} \bar{\Gamma}_1 \not{p} + \Gamma_1 \not{p} \bar{\Gamma}_2 \not{p} + \Gamma_2 \not{p} \bar{\Gamma}_1 \not{p}$$
$$+ \Gamma_2 \not{p} \bar{\Gamma}_2 \not{p}$$

$$p(\gamma^0 - \gamma^3)$$

$\cancel{\Gamma_i \not{p} \not{p} \Gamma_j \not{p}}$

$\neq$

$$\not{p}_1 \bar{\Gamma}_j = \not{p}\,(p_1 \mathcal{E}(a_j)\not{\varepsilon}(b_j) + \not{\varepsilon}(b_j)\not{\varepsilon}(a_j)\not{p}(4))$$

$$= p_1 \mathcal{E}(a)\,(\mathcal{E}(b)p_1 - \not{\varepsilon}(b)\not{p}_1) + \underbrace{\mathcal{E}(b)p_1 - \not{\varepsilon}(b)\not{p}_1)\not{\varepsilon}(a)\not{p}(4)}$$

$$= \mathcal{E}(b)p_1 \not{\varepsilon}(a)\not{p}(4)$$
$$- \not{\varepsilon}(b)(\mathcal{E}(a)p_1 - \not{\varepsilon}(a)\not{p}_1)\not{p}(4)$$

$$= \mathcal{E}(b)p_1 \not{\varepsilon}(a)\not{p}(4)$$
$$- \not{\varepsilon}(b)\mathcal{E}(a)p_1 \not{p}(4)$$
$$+ \not{\varepsilon}(b)\not{\varepsilon}(a)\not{p}_1\not{p}_4$$

$$= \mathcal{E}(b)\, p_1 \not{\varepsilon}(a)\not{p}(4)$$
$$- \not{\varepsilon}(b)\mathcal{E}(a)p_1\not{p}(4)$$
$$+ \not{\varepsilon}(b)\not{\varepsilon}(a)\; p_1 \cdot p_4$$
$$- \not{\varepsilon}(b)\not{\varepsilon}(a)\not{p}_4\not{p}_1$$

$$= \cancel{p_1 \cdot \mathcal{E}(a)\,(p_1\mathcal{E}(b))}$$

$$(p_1\cdot\mathcal{E}(a))(p_1\mathcal{E}(b)) - (p_1\mathcal{E}(a))\underset{\text{but wrg}}{\not{\varepsilon}(b)\not{p}_1} + p_1\cdot\mathcal{E}(b)\not{\varepsilon}(a)\not{p}(4) - (p_1\mathcal{E}(a))\not{\varepsilon}(b)\not{p}(4)$$

$$+ p_1 p_4 \not{\varepsilon}(b)\not{\varepsilon}(a) - \not{\varepsilon}(b)\not{\varepsilon}(a)\not{p}_4\not{p}_1$$

$$\not{p}_1 = 0$$
$$\not{p}_1\not{p}_1 = p_1^2 = 0$$

$\boxed{25}$ Nutze $\sum \varepsilon_i^{(\lambda)\mu} \varepsilon_j^{*(\lambda)\nu} = \cancel{\delta_{ij} - \hat{p}_i \hat{p}_j} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$

24.3

$\Rightarrow \sum_{\lambda} \varepsilon_\lambda^\mu \varepsilon_\lambda^{*\nu} = -g^{\mu\nu}$!! $= \delta^{\mu\nu}$

(Eichinvar!)

↳ von Seite 20 → Moseler + cos trick

$\sum_{\lambda_1 \lambda_2 s_1 s_2} \frac{|M|^2}{4} = \left(\frac{e g}{4 \cdot 2 p}\right)^4 \frac{1}{4} \sum_{\lambda_1 \lambda_2} tr\left(\left(\frac{\Gamma_1}{s^2} + \frac{\Gamma_2}{t^2}\right) \not p_2 \left(\frac{\bar\Gamma_1}{s^2} + \frac{\bar\Gamma_2}{t^2}\right) \not p_1 \right)$ $= E_{cm}$ I

I: $\sum_{\lambda_1 \lambda_2} tr(\Gamma_i \not p_2 \bar\Gamma_j \not p_1) = \sum_{\lambda_1 \lambda_2} tr\left(\left[p_1 \cdot \varepsilon^*(a_i) \not\varepsilon^*(b_i) \not p_2 + \not p(a_i) \not\varepsilon^*(a_i) \not\varepsilon^*(b_i) \not p_2\right] \cdot \left[p_1 \varepsilon(a_j) \not\varepsilon(b_j) \not p_1 + \not\varepsilon(b_j) \not\varepsilon(a_j) \not p(a_j) \not p_1\right]\right)$

$= \sum_{\lambda_1 \lambda_2} tr\big(p_1 \varepsilon^*(a_i) \not\varepsilon^*(b_i) \not p_2 (p_1 \varepsilon(a_i)) \not\varepsilon(b_j) \not p_1$

$+ p_1 \varepsilon^*(a_i) \not\varepsilon^*(b_i) \not p_2 \not\varepsilon(b_j) \not\varepsilon(a_j) \not p(a_j) \not p_1$

$+ \not p(a_i) \not\varepsilon^*(a_i) \not\varepsilon^*(b_i) \not p_2 (p_1 \varepsilon(a_j)) \not\varepsilon(b_j) \not p_1$

$+ \not p(a_i) \not\varepsilon^*(a_i) \not\varepsilon(b_i) \underbrace{\not p_2 \not\varepsilon(b_j)} \not\varepsilon(a_j) \not p(a_j) \not p_1 \big)$

$\downarrow 0$ da $p_1^2 = 0$

↳ $i=j$: $= tr(\not p_1 \not p_2 \not p_1 \not p_1) + tr(\not p_1 \gamma_\nu \not p_2 \gamma^\nu \not p(a_j) \not p_1)$

$+ tr(\not p(a_i) \not p_1 \gamma_\nu \not p_2 \gamma^\nu \not p_1) + tr(\not p(a_i) \gamma_\nu \gamma_\mu \not p_2 \gamma^\mu \gamma^\nu \not p(a_j) \not p_1)$

$\gamma_\mu \not a \gamma^\mu = -2 \not a$ $\quad = -2\, tr(\not p_1 \not p_2 \not p(a_j) \not p_1)$ $-2\, tr(\not p(a) \not p_1 \not p_1 \not p_1)$ $= 0$

$+ 4\, tr(\not p(a) \not p_1 \not p(a) \not p_1) = 4\, tr(\not p(a) \not p_1 \not p(a) \not p_1)$!!

$= 8\left((p(a) p_1)^2 - p(a)^2 p_1^2 + (p(a) p_1)^2\right)$

$= 8\left(2 (p(a) p_1)^2 - \underset{=0}{p(a)^2 p_1^2}\right) = 16\, (p(a) p_1)^2$

$\quad p^2(1 \pm \cos\theta)^2$

$i \neq j$: $= tr(\not p_1 \not p_1 \not p_1 \not p_1) + tr(\not p_1 \gamma_\nu \not p_1 \gamma^\mu \not p(a_j) \not p_1)$

$+ tr(\not p(a_i) \gamma_\mu \not p_1 \not p_1 \gamma^\mu \not p_1) + tr(\not p(a_i) \gamma_\mu \gamma_\nu \not p_1 \underbrace{\gamma^\mu \gamma^\nu}_{2 g^{\mu\nu} - \gamma^\nu \gamma^\mu} \not p(a_j) \not p_1)$

$\gamma_\mu \gamma^\mu = 4 \mathbb{1}$

$= 8\, tr(\not p(a_i) \not p_1 \not p(a_j) \not p_1) - 4\, tr(\not p(a_i) \not p_1 \not p(a_j) \not p_1)$!!

$= 4\, tr($ "

2.4.3 | 26

$(P(a_i)P_1)(P(a_j)P_1) - (P(a_i)P(a_j))(\underbrace{P_1^2}_{=0}) + (P_1 \cancel{\emptyset}) P(a_i)) P_1 P(a_j))$

$= 8\,((P(a_i)P_1)(P(a_j)P_1) - (P(a_i)P(a_j))(1+\cos(\theta)))$

$= 16\,(P(a_i)P_1 \cdot P(a_j)P_1) = 16\,p^4 \{ \frac{(1-\cos(\theta))(1+\cos(\theta))}{4 s'^2 c'^2}$

$(1+2)\,p\,(\cdot\bar{1}+\bar{2})\,p = 1p\bar{1}p + 1p\bar{2}p + 2p\bar{1}p + 2p\bar{2}p$

$= 16p^4\left(\frac{(P(4)P_1)^2}{s'^4} + \frac{2p^4(1+1)(1-\dots)}{s'^2c'^2} + \frac{(P(3)P_1)^2}{c'^4}\right)$

$= 16p^4\left(\frac{(1-\cos\theta)^2}{s'^4} + \frac{8\cancel{p^4}\, s'^2 c'^2}{s'^2c'^2} + \frac{(1+\cos\theta)^2}{c'^4}\right)$

$\cancel{= 16p^4(}$

falsch da $p_1 = -p_2$!

$i=j$: $= tr(\cancel{p}\gamma^\mu\, p_1^2\, \gamma_\nu \not{p}_2 \gamma^\nu \not{p}_1) + tr(\gamma_\nu \not{p}_2 \gamma^\nu \not{p}_1 \not{P}(a_j) \not{p}_1)$

$+ tr(\not{P}(a_i)\not{p}_1 \gamma_\nu \not{p}_2 \gamma^\nu \not{p}_1) + tr(\not{P}(a_i)\gamma_\mu \gamma_\nu \not{p}_2 \gamma^\nu \gamma^\mu \not{P}(a_j)\not{p}_1)$

$= -2\, tr(\not{p}_2 \not{p}_1 \underbrace{\not{P}(a_j)\not{p}_1}) \;\; -2\, tr(\not{P}(a_i)\not{p}_1\not{p}_2\not{p}_1)$

$\not{p}(2P(a_j)P_1 - \not{p}_1\not{P}(a_j))$

$= tr(\not{p}_2 \underbrace{\not{p}_1 \not{p}_1}_{0} \not{P}(a_j)) + P(a_j)P_1\, tr(\not{p}_2\not{p}_1)$

$+4\, tr(\not{P}(a_i)\not{p}_2\, \not{P}(a_j)\not{p}_1)$

$= -16\,(P(a_j)P_1)(\overset{2p^2}{P_2 \cdot P_1}) - 16\,(P(a_j)P_1)\,2p^2 + 4\,((P(a)P_2)(P(a)P_1)$

$+ - 0 + (P(a)P_1)(P(a)P_2))$

$= -32\,(2p^2(P(a)P_1)) + 8\,((P(a)P_2)(P(a)P_1))$

$\underline{27}|$ ~~$i\neq j$ = tr(~~

24.3

$a_1 = 4 \quad b_1 = \cancel{4}\,3$

$\Gamma_1 = \not\epsilon^*(4)(\not p_1 - \not p_4)\not\epsilon^*(3)$ $\quad \Gamma_2 = \not\epsilon(3)^*(\not p_1 - \not p_3)\not\epsilon^*(4)$

$\bar\Gamma_1 = \not\epsilon(3)(\not p_1 - \not p_4)\not\epsilon(4) \qquad \bar\Gamma_2 = \ldots$

$\boxed{i=j}$

$\sum_{\Gamma_1,\Gamma_2} tr(\Gamma_i \not p_2 \Gamma_j \not p_1) = tr(\gamma_\mu(\not p_1 - \not p(a_i))\gamma_\nu \not p_2 \gamma^\nu(\not p_1 - \not p(a_j))\gamma^\mu \not p_1)$

$\leftarrow \not\epsilon^*(a_i)(\not p_1 - \not p(a_i))\not\epsilon^*(b_i)\not p_2 \not\epsilon(b_j)(\not p_1 - \not p(a_j))\not\epsilon(a_j)\not p_1$

$\gamma_\nu \not a \gamma^\nu = -2a$

$= -2\, tr(\gamma_\mu(\not p_1 - \not p(a_i))\not p_2(\not p_1 - \not p(a_j))\gamma^\mu \not p_1)$

$\gamma_\nu \not a \not b \not c \gamma^\nu = -\not c \not b \not a$

$= 4\, tr((\not p_1 - \not p(a))\not p_2(\not p_1 - \not p(a))\not p_1)$

$= 4[\underbrace{tr(\not p_1 \not p_2 \not p_1 \not p_1)}_{=0} - \underbrace{tr(\not p_1 \not p_2 \not p(a)\not p_1)}_{=0} - \underbrace{tr(\not p(a)\not p_2 \not p_1 \not p_1)}_{=0}$

$+ tr(\not p(a)\not p_2 \not p(a)\not p_1)]$

$= 16[(p(a)p_2)(p(a)p_1) - 0 + (p(a)p_1)(p(a)p_2)] = 32[(p(a)p_2)(p(a)p_1)]$

$i \neq j$

$= tr(\gamma_\mu(\not p_1 - \not p(a_i))\gamma_\nu \not p_2 \gamma^\mu(\not p_1 - \not p(a_j))\gamma^\nu \not p_1)$

$= \gamma_\nu p(2)_\sigma \gamma^\sigma \gamma^\mu$

$= \gamma_\nu p(2)_\sigma (2g^{\mu\sigma} - \gamma^\mu \gamma^\sigma)$

$= 2\gamma_\nu p(2)^\mu - \gamma_\nu \gamma^\mu p(2)_\sigma \gamma^\sigma$

$= 2\gamma_\nu p(2)^\mu - g_{\nu\delta}\, \underbrace{\gamma^\delta \gamma^\mu}_{2g^{\delta\mu} - \gamma^\mu\gamma^\delta} \gamma^\sigma p(2)_\sigma$

$= 2\gamma_\nu p(2)^\mu - 2\underbrace{g_{\nu\delta}g^{\delta\mu}}_{\delta_\nu^\mu}\gamma^\sigma p(2)_\sigma + \gamma^\mu \gamma_\nu \gamma^\sigma p(2)_\sigma$

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$$tr \ldots = 2tr(\not{p}_2(\not{p}_1-\not{p}(a_i))\gamma_\nu(\not{p}_1-\not{p}(a_j))\gamma^\nu\not{p}_1)$$

$$-2\,tr(\gamma_\mu(\not{p}_1-\not{p}(a_j))\not{p}_2(\not{p}_1-\not{p}(a_j))\gamma\not{p}_1)$$

$$+tr(\gamma_\mu(\not{p}_1-\not{p}(a_i))\gamma^\mu\gamma_\nu\not{p}_2(\not{p}_1-\not{p}(a_j))\gamma^\nu\not{p}_1)$$

$$= -4\,tr(\not{p}_2(\not{p}_1-\not{p}(a_i))(\not{p}_1-\not{p}(a_j))\not{p}_1)$$

$$+4\,tr((\not{p}_1-\not{p}(a_i))\not{p}_2(\not{p}_1-\not{p}(a_i))\not{p}_1)$$

$$\cancel{+4}tr((\not{p}_1-\not{p}(a_i))\not{p}_2(\not{p}_1-\not{p}(a_j))\not{p}_1)$$

$$-8$$

$$\not{p}_1^2=0$$

$$= +4tr(\not{p}_2(\not{p}_1-\not{p}(a_i))\not{p}(a_j)\not{p}_1)$$

$$-4\,tr((\not{p}_1-\not{p}(a_j))\not{p}_2\,\not{p}(a_i)\not{p}_1)$$

$$+8tr((\not{p}_1-\not{p}(a_i))\not{p}_2\not{p}(a_j)\not{p}_1)$$

$$= \not{p}_2\not{p}_1\not{p}(a_j)\not{p}_1$$
$$-\not{p}_2\not{p}(a_i)\not{p}(a_j)\not{p}_1$$
$$= \not{p}_2\not{p}_1\cdot 2\,p(a_j)p_1$$
$$-\ldots$$

$$= +4\,tr(\not{p}_2(\not{p}_1-\not{p}(a_i))\not{p}(a_j)\not{p}_1)+4\,tr(\not{p}(a_j)\not{p}_2\not{p}(a_i)\not{p}_1)$$

$$-8\,tr(\not{p}(a_i)\not{p}_2\not{p}(a_j)\not{p}_1)$$

$$= 8\,p(a_j)p_1\,tr(\not{p}_1\not{p}_2)-4tr(\not{p}_2\not{p}(a_i)\not{p}(a_j)\not{p}_1)$$
$$+4tr(\not{p}(a_j)\not{p}_2\not{p}(a_i)\not{p}_1)-8\,tr(\not{p}(a_i)\not{p}_2\not{p}(a_j)\not{p}_1)$$

$$= 32\,(p(a_j)p_1)(\overset{2p^2}{p_1p_2})-16\,[(p_2p(a_i))(p_1p(a_j))-(p(a_i)p(a_j))(p_1p_2)$$
$$+((p(a_i)p_1)(p_2p(a_j))]$$

$$+16\,[(p(a_j)p_2)(p(a_i)p_1)-(p(a_i)p(a_j))(p_1p_2)+(p_1p(a_j))(p_2p(a_i))]$$

$$-32\,((p(a_i)p_2)(p(a_j)p_1)-(p(a_i)p(a_j))(p_1p_2)+(p(a_i)p_1)(p(a_j)p_2))$$

$$= 32\,(p(a_j)p_1)\cdot 2p^2-32\,(\qquad ||$$

$\overline{ZJ \,|\, Z4.3}$ ~~$1p_2 1p_1 + 1p_2 2p_1$~~ $11 + 12 + 21 + 22$

$= 32 p^4 \Big[\frac{(1-c)(1+c)}{s'^4}$

$p_1 = p\begin{pmatrix}1\\s\\0\\c\end{pmatrix}$ $p_2 = p\begin{pmatrix}1\\-s\\0\\-c\end{pmatrix}$

$+ \frac{(2(1+c) - (1+c)(1+c) + 4 - 2 \cdot (1\mp c)(1\mp c))}{s'^2 c'^2}$

$p_3 = p\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$ $p_4 = p\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$

$+ \frac{(2(1-c) - (1-c)^2 + 4 - (1+c)^2)}{s'^2 c'^2}$

$+ \frac{(1-c)(1+c)}{c'^4} \Big]$

$+ 4(1+c^2)$

~~$= 32p^4 \,|\, 2(1-c)(1+(1-c^2) + 4$~~

$32p^4 \,[\, (1-c^2)\left(\frac{1}{s'^4} + \frac{1}{c'^4}\right) + \frac{1}{s'^2 c'^2}\left(8 - \underbrace{2(1-c)^2}_{8 s'^4} - \underbrace{2(1+c)^2}_{8 c'^4}\right)$

$4 s'^2 c'^2$

$= \frac{8}{s^2}(1-c^2)$

$= 4\left(\frac{c'^2}{s'^2} + \frac{s'^2}{c'^2}\right)$

$= 32 p^4 \left(4(\cot'^2 + \tan'^2) + \frac{8}{s'^2 c'^2} - \frac{8 s'^2}{c'^2} - \frac{8 c'^2}{s'^2}\right.$

$8\frac{s'^2 + c'^2}{s'^2 c'^2}$

$= 32 p^4 \left(4(\cot'^2 + \tan'^2)\right) + \frac{16}{s^2}(1-c^2) \,)$

$= 32 p^4 (1-c^2)\left(\frac{1}{s'^4} + \frac{1}{c'^4} + \frac{4}{s'^2 c'^2}\right) = 32 p^4 8 (1-c^2)\left(\frac{2}{s^4} + \frac{1}{s^2}\right)$

$= 8 \cdot 32 p^4 \frac{(1-c^2)(2 + s^2)}{s^4}$

$\left(\left(\frac{1}{s'^2} + \frac{1}{c'^2}\right)^2 + \frac{1}{s'^2 c'^2}\right)$

$= \frac{1}{s'^4 c'^4} + \frac{2}{s'^2 c'^2}$

$\frac{16}{s^4} \qquad \frac{8}{s^2}$

$= 8 \cdot 32 p^4 \frac{(1-c^2)(2+s^2)}{s^4}$

$\boxed{30 \; 124.3}$ $\langle M \rangle = \frac{1}{4} \frac{(Qy)^4}{(2\pi)^4} 2 \cdot 8 \cdot 2 \cancel{(2\pi)^4} \frac{(1-c^2)(2+s^2)}{s^4}$

$\cancel{= 4 \int}$

$= 4 \, (Qy)^4 \, \frac{(1-\cos^2\theta)(2+\sin^2\theta)}{\sin\theta^4}$ ($1-\cos^2\theta = \sin^2(\theta)$) $= 4 \, (Qy)^2 \frac{(2+\sin^2\theta)}{\sin^2\theta}$

$= \frac{4(Qy)^2}{(1+2\cosh^2(\eta)}$

$\int d\Omega = \int 2\pi \int d\theta x \; 4 (Qy)^4 \; \frac{\cancel{(1-x^2)}(2+1-x^2)}{(1-x^2)^2}$

$\sin^2 x = 1 - \cos^2(x)$

$\frac{(2+1}{} \frac{(3-x^2)}{1-x^2}$

$= \frac{3}{1-x^2} - \frac{x^2}{1-x^2}$

$3 \, \text{artanh}(x) \Big|_{x_1}^{x_2}$

$\int dx \frac{x^2}{1-x^2} = x^2 \text{artanh}\, x - 2 \int \frac{x}{1-x^2} dx$

$= x^2 \text{artanh}\, x - \int_{x_1}^{x_2} dx \frac{1}{1-x}$

$= x^2 \text{artanh}\, x + \ln|1-x^2| \Big|_{x_1}^{x_2}$

$\int d\theta \left(\frac{2}{\sin^2(\theta)} + 1 \right) \sin\theta$

$\int dx \frac{2}{1-x^2} + \int d\theta \, \sin\theta$

$= 2(\text{artanh}(\cos\theta_2) - \text{artanh}(\cos\theta_2)$

$- \cos\theta_2 + \cos\theta_1$

$\int d\theta \frac{2}{\sin\theta} + \sin\theta$ $\quad x = \cos\theta$, $dx = - d\theta \sin\theta$

$= -\cos\theta \Big|_{\theta_1}^{\theta_2} + 2 \int_{\theta_1}^{\theta_2} d\theta \, \sin\theta \frac{1}{1-\cos^2\theta}$

$= 2 \int_{x_2}^{x_1} dx \frac{1}{1-x^2}$

$2 \, \text{artanh}(\cos\theta_1) - 2 \, \text{ath}(\cos\theta_2)$

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↳ Golden Rule $\nearrow p_f/p_i$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|M|^2}{E_{sp}^2} = \frac{1}{(8\pi E_{sp})^2} |M|^2 = \left(\underbrace{\frac{(gQ)^2}{2\pi E_{sp}}}_{F}\right)^2 \underbrace{\frac{2+\sin^2\theta}{\sin^2\theta}}_{= 2+}$$

$\eta \in -2.5 < 2.5$

$$\Rightarrow \sigma = 2\pi F \left(3\,\text{artanh}(x)\Big|_{\cos(\theta_1)}^{\cos(\theta_2)} - x^2\,\text{artanh}(x) + \ln(1-x^2) \right)$$

$$= 2\pi F \left((3-x^2)\,\text{artanh}(x) + \ln(1-x^2) \right)$$

$$= 2\pi F \left((3 - \cancel{\cos\theta_2})\,\text{artanh}(\cos\theta_2) - (2+\sin^2\theta_1)\,\text{artanh}(\cos\theta_1) \right.$$

$$2 + \frac{\cos}{\sin^2\theta_2}$$

$$\left. + \underbrace{2\ln\left(\frac{\sin\theta_1}{\sin\theta_2}\right)}_{= 2(\text{artanh}(\cos\theta_2) - \text{artanh}(\cos\theta_1))} \right)$$

$\cancel{= 2\pi F}$ $-\Delta\theta$

$\eta = -\ln(\tan\frac{\theta}{2})$

$e^{-\eta} = \tan\frac{\theta}{2}$

$$\sin x = \frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})} = \frac{2e^{-\eta}}{1+e^{-2\eta}} = \frac{2}{e^{\eta}+e^{-\eta}} = \frac{1}{\cosh(x)}$$

$$\cos x = \frac{1-e^{-2\eta}}{1+e^{-2\eta}} = \tanh\eta$$

$$\Rightarrow \sigma = 2\pi F \cdot \left(2(\tanh\eta_1 + \tanh\eta_2) + \eta_1 - \eta_2\right)$$

$-2.5 \ldots 2.5$

$$\stackrel{!}{\approx} 2\pi F \cdot (8.94)$$

$$= \frac{1}{2\pi} \left(\frac{(gQ)^2}{E_{sp}}\right)^2 8.94$$